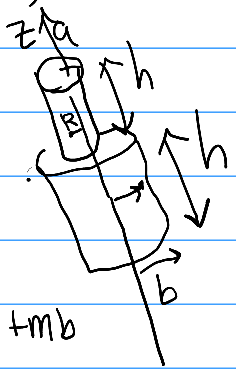
Quarta, 1 de abril

(5)




$$I_z = \int_{vol.} R^2\,dm$$

$$= \int_{V_1} R^2\,dm + \int_{V_2} R^2\,dm$$

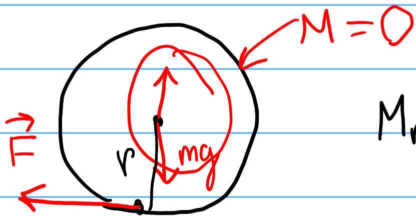
$$I_z = I_1 + I_2 \qquad I_{cil} = \frac{1}{2} m r^2$$

$$\begin{cases} m = m_a + m_b \\ \dfrac{m_a}{\pi a^2 h} = \dfrac{m_b}{\pi b^2 h} \end{cases}$$

$$I_z = \frac{m_a}{2} a^2 + \frac{m_b}{2} b^2 \qquad m_a, m_b \rightarrow m$$

(11) (C)

$$\alpha = \frac{\sum_i M_i}{I}$$


$$M_r = Fr \qquad \alpha = \frac{Fr}{I}$$

(a) $\alpha = \dfrac{d\omega}{dt}$ $\qquad \omega_0 = \dfrac{d\theta}{dt} = \dfrac{\Delta\theta}{\Delta t_1}$ (constante)

$10 \times 2\pi$

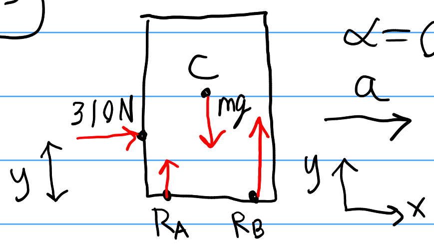
$\alpha$ = constante $\qquad \omega_f = 0$

$$\Rightarrow \alpha = \frac{\Delta\omega}{\Delta t} = \frac{\omega_f - \omega_0}{\Delta t_2}$$

(b) $\alpha = \omega \dfrac{d\omega}{d\theta}$ $\qquad \Delta\theta = \dfrac{\omega_0^2}{2\alpha}$

$$\alpha \int_0^{\Delta\theta} d\theta = \int_{\omega_0}^{0} \omega\,d\omega = -\frac{\omega_0^2}{2}$$

(3)




$\alpha = 0$

$a \rightarrow$

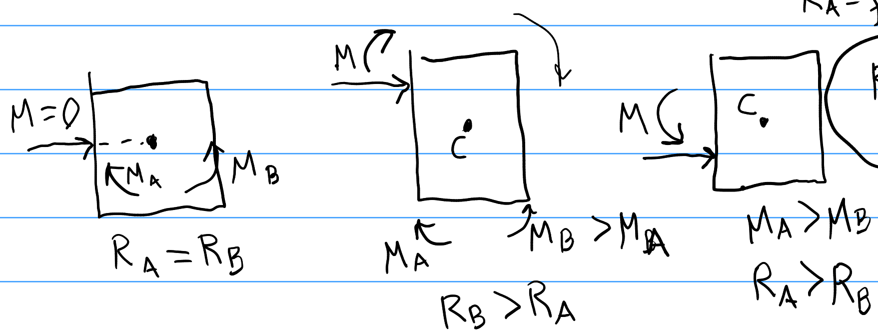
$$\begin{cases} \sum_i F_x = ma \\ \sum_i F_y = 0 \\ \sum_i M_c = 0 \end{cases}$$

4 incógnitas

$R_A, R_B, a, y$

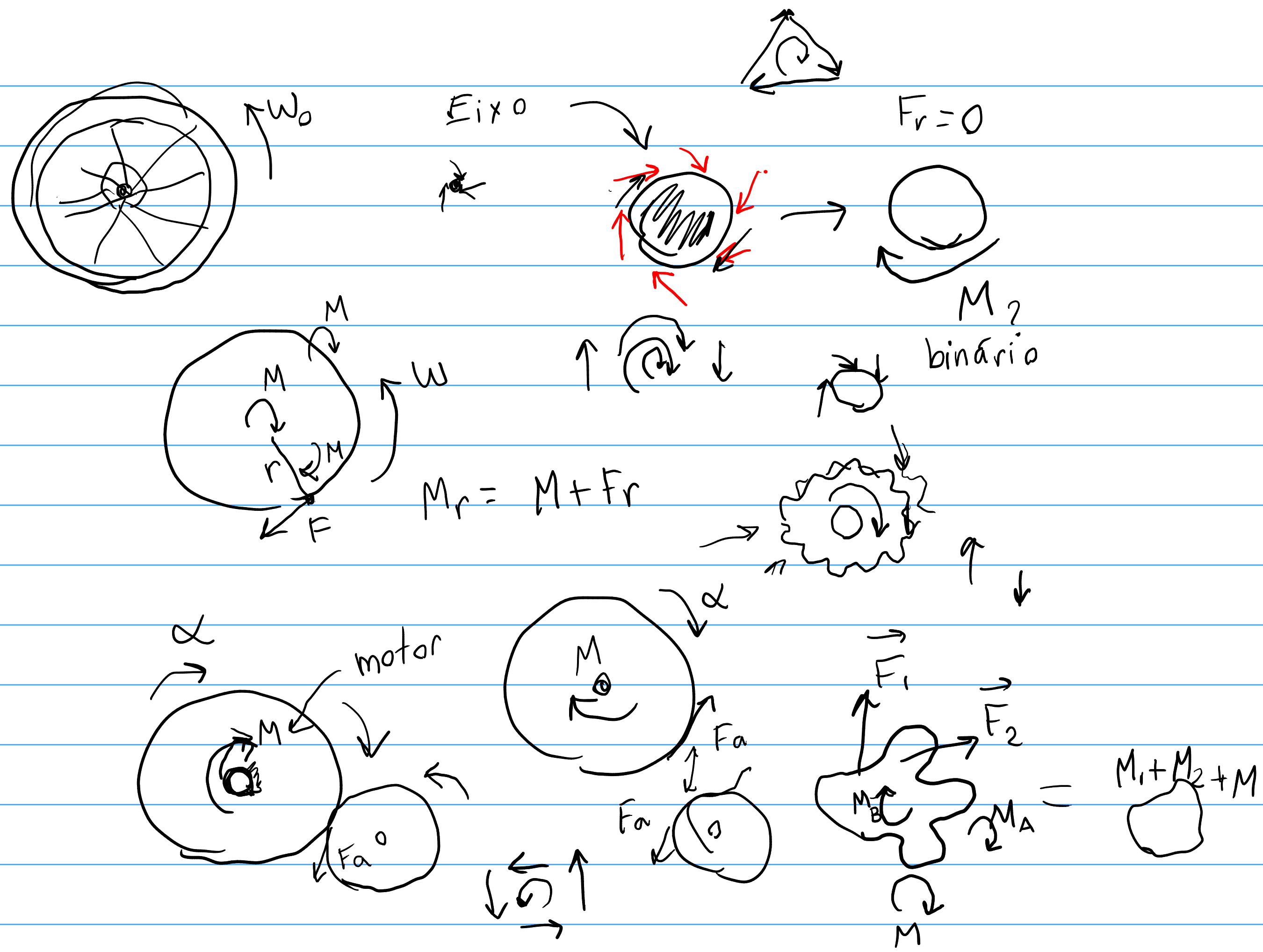
parâmetro

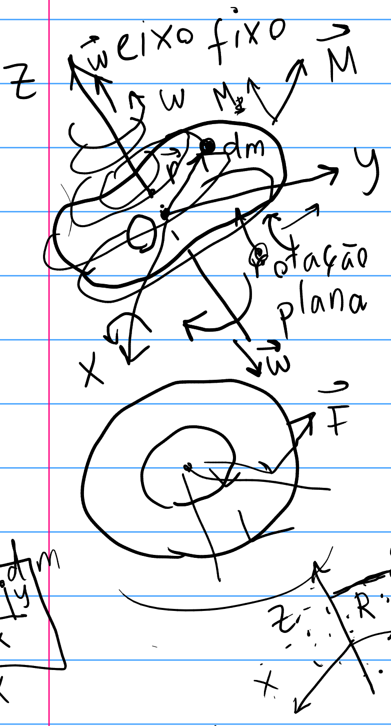
$R_A = f_A(y) \qquad R_B = f_B(y)$




$R_A \geq 0$

$R_B \geq 0$




dm → $\vec r = x\hat\imath + y\hat\jmath + z\hat k$

$$d\vec f = \vec a\, dm$$

$\vec a = ?$ $\quad \vec\omega = \omega \hat k \quad \vec\alpha = \alpha \hat k$

$$\vec v = \vec\omega \times \vec r$$

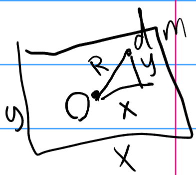
$$\vec a = \vec\alpha \times \vec r + \vec\omega \times \vec v$$

$$= \begin{vmatrix} \hat\imath & \hat\jmath & \hat k \\ 0 & 0 & \alpha \\ x & y & z \end{vmatrix} + \vec\omega \times \begin{vmatrix} \hat\imath & \hat\jmath & \hat k \\ 0 & 0 & \omega \\ x & y & z \end{vmatrix}$$

$$= -\alpha \begin{vmatrix} \hat\imath & \hat\jmath \\ x & y \end{vmatrix} + \vec\omega \left( -\omega \begin{vmatrix} \hat\imath & \hat\jmath \\ x & y \end{vmatrix} \right)$$

$$= \alpha(x\hat\jmath - y\hat\imath) + \vec\omega\left(\omega(x\hat\jmath - y\hat\imath)\right)$$

$$= \alpha(x\hat\jmath - y\hat\imath) - \omega^2(x\hat\imath + y\hat\jmath)$$




$x^2 + y^2 = R^2$ $\quad d\vec f = \left(\alpha(x\hat\jmath - y\hat\imath) - \omega^2(x\hat\imath + y\hat\jmath)\right) dm$

$|d\vec M|_z = \cancel{d\vec f \times} \vec r = \vec r \times d\vec f|_z \quad d\vec M = \begin{vmatrix} \hat\imath & \hat\jmath & \hat k \\ x & y & z \\ -\alpha y - \omega^2 x & -\alpha x - \omega^2 y & 0 \end{vmatrix} dm$

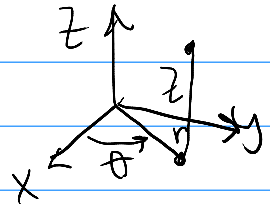
$$M_z = \int_{vol.} \alpha (x^2 + y^2)\, dm \quad I$$

$$I = \int R^2\, dm$$

# Exemplo 5.4


$$I_z = \int_{cil.} r^2 \, dm$$

$$dm = \rho \, d\text{volume.} \leftarrow dx\,dy\,dz$$



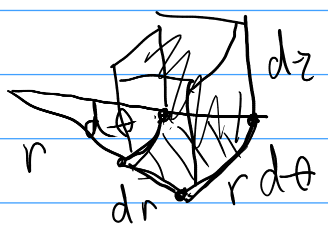

$$(x, y, z) \rightarrow (r, \theta, z)$$

$$r = \sqrt{x^2+y^2} \qquad \theta = \tan^{-1}\left(\frac{y}{x}\right)$$

$$\theta \rightarrow \theta + \Delta\theta$$

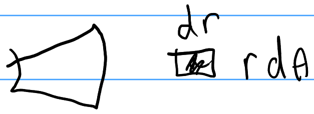
$$r \rightarrow r + dr$$

$$z \rightarrow z + dz$$




$$\text{base} \approx (r\,d\theta)\,dr$$

$$d\,\text{vol.} = r\,d\theta\,dr\,dz$$




$$I_z = \int_{vol} r^2 \left(\rho \, r\,d\theta\,dr\,dz\right)$$

$$= \rho \int_0^R \int_0^L \int_0^{2\pi} r^3 \, d\theta\,dz\,dr = \rho \cancel{2}\pi L \left(\frac{R^4}{\cancel{4}_{\,2}}\right)$$

$$m = \rho\left(\pi R^2 L\right) \qquad \Rightarrow \qquad I_z = \frac{1}{2} m R^2$$